Morphology with parabolic structuring elements

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Abstract

Morphological erosion and dilation filters employ a structuring function, with flat structuring functions being the most common example. Parabolic[1] structuring functions are less well known but theoretically very important and practically very useful. This paper briefly introduces morphology using parabolic structuring functions, describes the ITK classes used to implement them and includes a number of sample applications.

Contents

1 Introduction **1**

2 Brief background theory **2**

3 ITK classes **2**

4 Applications **4**
4.1 Sharpening . . . 4
4.2 Distance Tranforms . . . 4

5 Conclusions and further work **5**

1 Introduction

Parabolic structuring functions (PSF) have the following important properties:

[1] This article will use the term "parabolic", but much of the literature uses "quadratic"

- They are closed under dilation - i.e. dilating two PSFs results in a third PSF.
- An n-dimensional PSF can be obtained by combining n one-dimensional PSFs in independent directions.
- PSFs are rotationally symmetric allowing the dimensional decomposition by dilation.

These properties make PSFs the morphological counterpart of the Gaussian in linear image processing [1].

The dimensional decompostion properties lead to efficient algorithms for implementing parabolic morphological operations.

Parabolic morphology operations are useful in image sharpening, distance transforms, are a less vigorous alternative to conventional morphological operations based on flat structuring elements and a potentially useful, faster, alternative to shaped structuring elements such as the "rolling ball" often used in background estimation in tools such as ImageJ.

2 Brief background theory

A dilation by a one dimensional parabolic structuring element is illustrated in Figure 1. The dilation of the signal at point A is found by lowering the structuring function centred at A until the structuring function comes into contact with the signal, in this case at point B. The dilation at this point is given by point C, the height of the parabola turning point. The equivalent erosion is calculated by raising an inverted parabola into the signal from below.

3 ITK classes

The filters discussed in this section implement the "point of contact" algorithm [1]. Algorithms that are more efficient for larger kernel sizes are available and are discussed by van den Boomgaard et al, but haven't been implemented in this package. The classes *itkParabolicErodeImageFilter*, *itkParabolicDilateImageFilter*, *itkParabolicCloseImageFilter* and *itkParabolicOpenImageFilter* provide the range of parabolic morphology operations. These classes derive from *itkParabolicErodeDilateImageFilter* and *itkParabolicOpenCloseImageFilter*, which implement the core functionality. The open and close filters offer an optional border padding facility.

The controling methods available for these classes is :

- **Set/GetScale** : a PSF is usually defined as $f(x) = \frac{1}{2\rho}x^2$. This method sets ρ.
- **Set/GetUseImageSpacing** : defines whether the units for ρ are voxels or image spacing.
- **SetSafeBorder** : controls whether border padding is used in opening or closing operations.

Note that there are actually a variety of ways in which scale is of PSFs is defined in the literature, including $f(x) = (\frac{x}{scale})^2$ and $f(x) = (\frac{x^2}{scale})$. Logarithmic scales are another alternative.

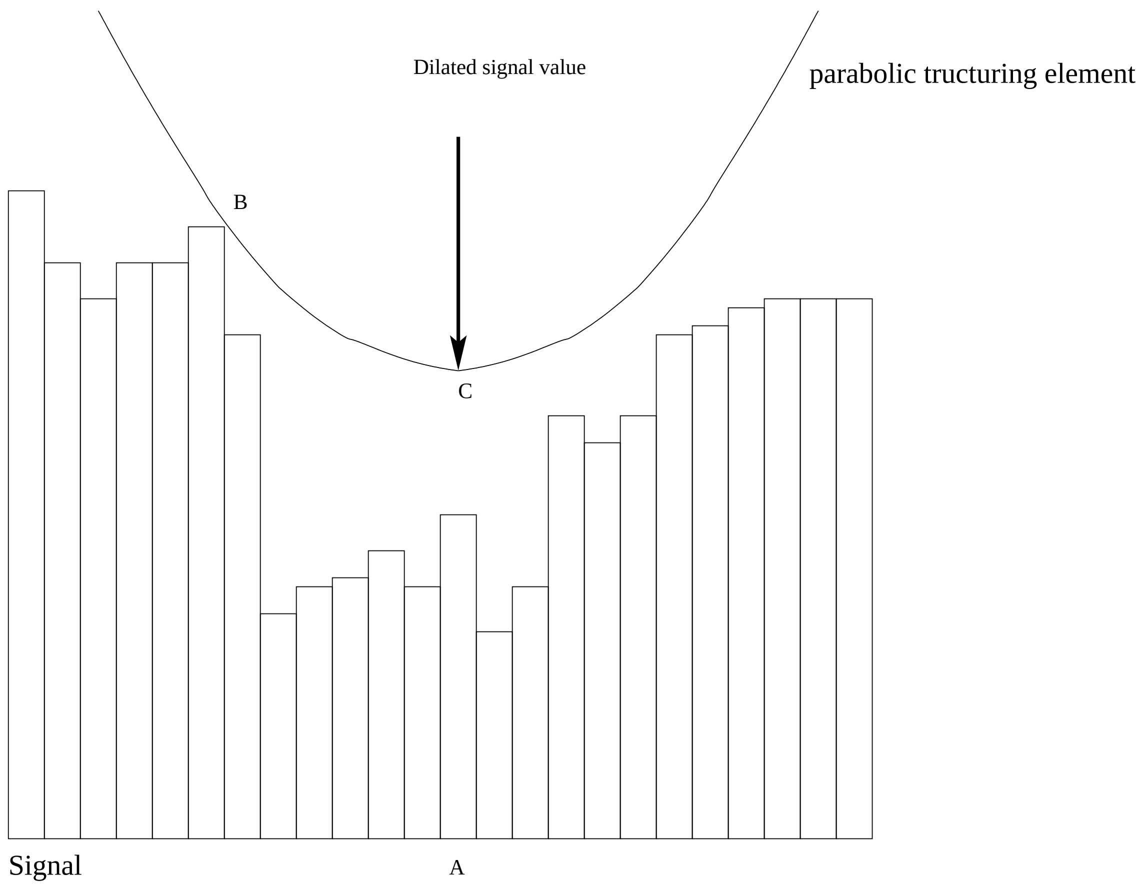

Figure 1: Dilation of a 1D signal with a parabolic structuring element.

4 Applications

4.1 Sharpening

Image sharpening using parabolic operations has been analysed in [3]. This form of sharpening is based on the following formula:

$$\varepsilon[f](x,\rho) = \begin{cases} F^{\oplus}(x,\rho), & F^{\oplus}(x,\rho) - F(x,0) < F(x,0) - F^{\ominus}(x,\rho), \\ F^{\ominus}(x,\rho), & F^{\oplus}(x,\rho) - F(x,0) > F(x,0) - F^{\ominus}(x,\rho), \\ F(x,0), & otherwise \end{cases} \quad (1)$$

where $F^{\oplus}(x,\rho)$ indicates dilation by parabolic structuring element with scale ρ amd $F^{\ominus}$ indicates erosion. In summary, the output of the sharpening process at a pixel is the dilation, if the difference between the dilation and the input is less than the difference between the erosion and the input. This formula describes one step in an iterative process. The parameters controling the process are scale and number of iterations.

This sharpening algorithm has been implemented in the *itkMorphologicalSharpeningImageFilter*, and *testSharpening* illustrates its use. Figure 2 illustrates the results of sharpening using profiles through an artificial image.

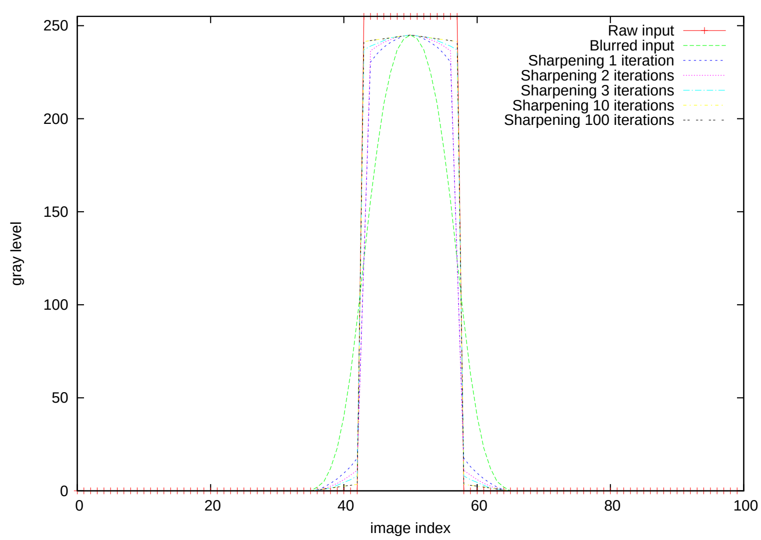

Figure 2: Profiles through an artificial image illustrating the sharpening process. Scale value was 1 pixel.

4.2 Distance Tranforms

Parabolic erosions and dilations can be used to construct distance transforms. Two classes have been provided in this package:

- *itkMorphologicalDistanceTransformImageFilter* for one sided distance transforms.
- *itkMorphologicalSignedDistanceTransformImageFilter* for signed distance transforms.

The concept is fairly simple - consider applying a parabolic erosion, with scale 1, to a mask image with values of 0 and inf. The value of the erosion at mask locations with value 0 will remain 0. The value at mask locations of inf are equal to the square of the distance to the nearest 0 value. The idea can be easily extended to allow computation of the inside and outside distances without rethresholding. This method should be more accurate than basic chamfer approaches and appears slightly faster than the *SignedDanielssonDistanceTransform* currently provided in ITK, as illustrated in Table 1. The parabolic version does not return voronoi tesselations or vectors pointing to the nearest inverse pixel because such things aren't required in construction of the distance transform. I cannot comment on the relative accuracy of the two methods at this stage as there appears to be no difference in output for the cthead image, but expect that the Danielsson method uses approximations that aren't necessary in the parabolic version. There are prospects for further improvements in performance when the parabolic methods are threaded. The parabolic method should retain efficiency in arbitary dimensions due to the separable nature of the parabolic operations.

Parabolic	Danielsson
0.0416	0.0588

Table 1: Execution times for computing signed distance tranforms using parabolic morphology and the Danielsson method in ITK. The standard, 256×256 *cthead* image was used and the test was repeated 100 times. These figures were obtained using perfDT, in this package.

5 Conclusions and further work

This article introduces implementations of an important class of morphological operations - erosions and dilations be parabolic structuring elements. Applications of these operations to sharpening and distance transforms are also presented. The latter appears to outperform some existing implementations, but more tests are necessary.

Acknowledgements

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References

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